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Optimal Design of Laminated Composite Plates in a Fuzzy Environment

Seung Jo Kim* and Nam Seo Goo†
Seoul National University, Seoul 151-742, Republic of Korea

The present study focuses on a fuzzy optimization approach to the optimal design of simply supported laminated composite plates. The goal is minimum weight design of the composite plates. The design constraints are the magnitude of center displacement and lamina failure criteria. A procedure of two-level optimization in a fuzzy environment is presented and this procedure is applied to the design of laminated composite plates to consider the influences of fiber direction and thickness of each lamina separately. It is shown that the membership function in the case of failure can be deduced from continuum damage mechanics. The membership function for failure criteria is the function of the internal state variable for degree of damage in the continuum damage mechanics. The modified Weibull membership function is chosen to be used as a simple case of damage progression. The finite element method is used to calculate the displacement of the plate and the stresses of each lamina. The fuzzy optimization formulation as a max-min problem is transformed into a classical one. This classical optimization problem is solved using the augmented Lagrange multiplier method with the quasi-Newton method. Two examples are solved to demonstrate the procedure. From the results, it is shown that optimal design of laminated composite plates is more useful in a fuzzy environment and the modified Weibull membership function characterizes the fuzziness of failure.

Nomenclature

b_i	= upper limit of the inequality constraint
$\tilde{D}$	= fuzzy decision
$f(X)$	= objective function
$\tilde{f}(X)$	= fuzzy goal
f^l	= lower bound of objective function
f^u	= upper bound of objective function
$\tilde{g}$	= fuzzy constraint
$g_i(X)$	= inequality constraint
g^l	= severe condition of constraint
g^u	= loose condition of constraint
S	= shear strength
s	= variable introduced in the formulation of fuzzy optimization
s^*	= maximum satisfaction
t	= thickness vector
u	= central displacement of the plate
$\bar{u}$	= upper limit of central displacement of the plate
X	= longitudinal strength
X	= design vector
X_t	= transition point from exponential function to linear function in modified Weibull function
X^*	= optimal design vector
Y	= transverse strength
$Z(\theta)$	= total objective function
$\mu_{\tilde{D}}$	= membership function of fuzzy decision
$\mu_{\tilde{f}}$	= membership function of fuzzy goal
$\mu_{\tilde{g}}$	= membership function of fuzzy constraint
$\mu_{\tilde{S}}$	= membership function of shear strength
$\mu_{\tilde{X}}$	= membership function of longitudinal strength
$\mu_{\tilde{Y}}$	= membership function of transverse strength
$\sigma_1, \sigma_2, \sigma_{12}$	= stresses in principal material coordinate

Superscripts

l	= lower bound or severe condition
u	= upper bound or loose condition
~	= fuzzifier
*	= optimal value

Introduction

MOST of the traditional tools for optimal design are crisp and deterministic. In other words, a candidate for the optimum solution is checked against the constraints and is determined as feasible or not. This means that our decision tools for optimal design are dichotomous. However, in the real world design decisions are redundant and graded. Fuzzy set theory was created with this concept in mind and has opened doors to "more or less" type decisions which are closer to human thinking and feeling. Since it was first proposed by Zadeh,[1] this theory has been developed mathematically, and in recent years, applied in a wide variety of scientific areas, such as artificial intelligence, robotics, image processing, speech recognition, management science, expert systems, logic, control engineering, and decision theory. It is noted that fuzzy theory does not describe certain things fuzzily, it describes fuzzy things in certain terms.

In the area of structural optimization, Rao[2,3] applied the fuzzy theory to practical structural optimization problems. He showed that fuzzy optimization problems can be solved using an ordinary nonlinear programming technique. Xu[4] proposed the two-phase method for fuzzy optimization of structures. Yeh and Hsu[5] considered structural optimization with fuzzy parameters using the concept of fuzzy expected cost and possibility theory, similar to probability theory. Kim et al.[6] presented two-level optimization in a fuzzy environment and applied it to the optimal design of composite plates with linear membership function. Dhingra et al.[7] applied fuzzy mathematical programming techniques to multiple-objective design problems and discussed the impact of several nonlinear membership functions on the overall design process. Rao et al.[8] presented a fuzzy nonlinear goal programming approach for solving multiobjective optimization problems.

In this paper, the two-level optimization procedure in a fuzzy environment is presented in a more general form and it

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*Associate Professor, Department of Aerospace Engineering. Member AIAA.

†Graduate Research Assistant, Department of Aerospace Engineering.

is shown that the membership function for failure can be deduced from the theory of continuum damage mechanics. An optimal design of laminated composite plates with this membership function is attempted. The design goal is to minimize the weight of the plate. The design variables are the thicknesses and the fiber directions of the composite plate. The design constraints are the magnitude of the center displacement and lamina failure criteria of the simply supported plate. Maximum stress theory is used as a failure criteria of the anisotropic materials. First ply failure is adopted as the failure criteria of the laminated composite plate. To calculate the displacement of the plate and the stresses of each lamina, an analysis program is developed using the finite element method. Two examples are considered to demonstrate the optimal design procedure of laminated composite plates using the fuzzy concept.

Two-Level Optimization in a Fuzzy Environment

An optimization problem in a fuzzy environment can be formulated as the following max-min one with ordinary constraints:

$$\max_X \min_{j=1,\ldots,l+n} \left[\mu_{\tilde{D}_j}(X)\right]$$

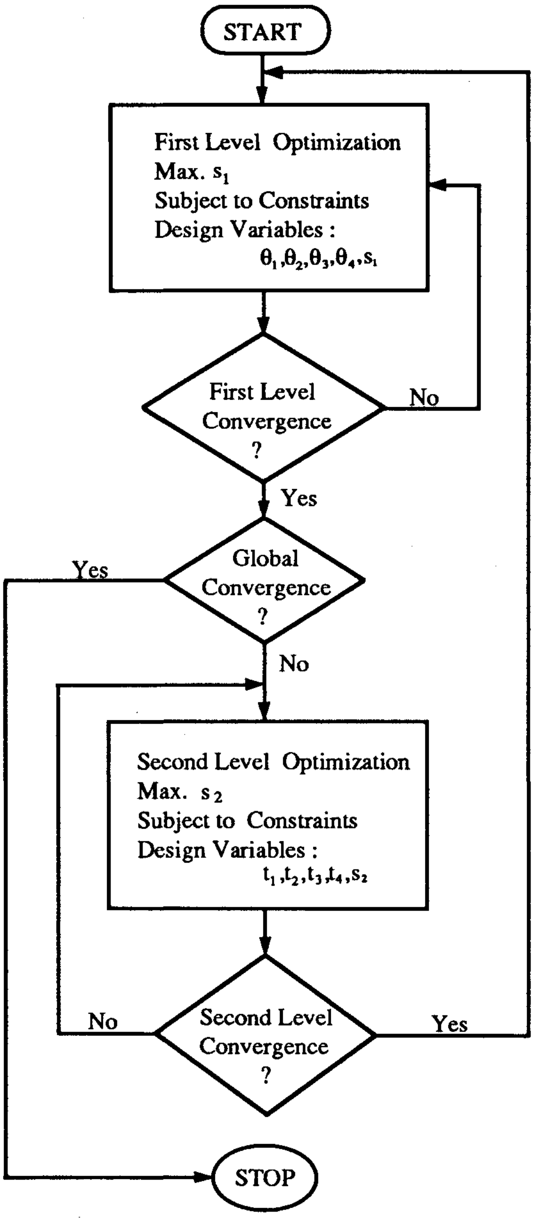

Fig. 1 Two-level optimization procedure in a fuzzy environment.

subject to

$$g_i(X) \leq b_i, \qquad i = 1, 2, \ldots, m - l \tag{1}$$

Of the m constraints, l constraints are in a fuzzy environment and the others are not. Equation (1) can be transformed into the following:

Find X and s such that s is maximized subject to

$$s - \mu_{\tilde{D}_j}(X) \leq 0, \qquad j = 1,2,\ldots,l + n$$

$$g_i(X) \leq b_i, \qquad i = 1,2,\ldots,m - l \tag{2}$$

Multilevel optimization techniques have been applied to many problems in structural design. In the case of the composite plate, two-level optimization techniques were used to avoid the introduction of an additional weighting function.[9,10] In a fuzzy environment, the new variable [s in Eq. (2)] makes the optimization procedure more complex. Therefore, this technique is implemented to the fuzzy optimization of composite plates and is used to reduce complexity. At the first level, the fiber directions of laminas are considered to be the only design variables. At this level, the optimal fiber angles which maximize the satisfaction s_1 of the first level are found. The goal of the second level is to find thicknesses which maximize the satisfaction s_2 of the second level.

This procedure can be formulated in the following mathematical form.

The first level: Find X which maximizes s_1 subject to

$$s_1 - \mu_{\tilde{D}_j}(X) \leq 0, \qquad j = 1,2,\ldots,l + n$$

$$g_i(X) \leq b_i, \qquad i = 1,2,\ldots,m - l \tag{3}$$

where

$$X^t = (\theta_1, \theta_2, \theta_3, \theta_4, s_1)$$

The second level: Find X which maximizes s_2 subject to

$$s_2 - \mu_{\tilde{D}_j}(X) \leq 0, \qquad j = 1,2,\ldots,l + n$$

$$g_i(X) \leq b_i, \qquad i = 1,2,\ldots,m - l \tag{4}$$

where

$$X^t = (t_1, t_2, t_3, t_4, s_2)$$

Figure 1 shows the two-level optimization procedure in a fuzzy environment.

Membership Function for Failure Criteria Through the Concept of Continuum Damage Mechanics

It is easily admitted that there is fuzziness in physical phenomena; however, the determination of membership function is difficult as Norwich[11] and Dombi[12] point out. Therefore, the most frequently used form is the linear one although the membership function derived from the probability density function is used sometimes.

The linear membership functions for the constraints can be written as

$$\mu_{\tilde{g}} = \begin{cases} 1 & g < g^l \\ 1 - (g - g^l)/(g^u - g^l) & g^l \leq g \leq g^u \\ 0 & g > g^u \end{cases} \tag{5}$$

Similarly, the membership functions of the objective functions are written as

$$\mu_{\tilde{f}} = \begin{cases} 1 & f < f^l \\ 1 - (f - f^l)/(f^u - f^l) & f^l \leq f \leq f^u \\ 0 & f > f^u \end{cases} \tag{6}$$

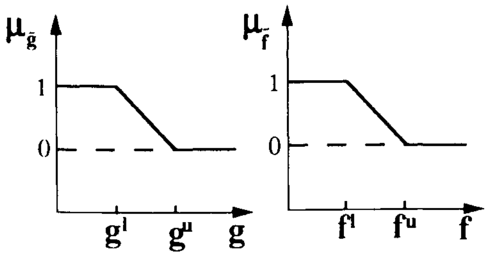

Fig. 2 Linear approximation of membership function: a) fuzzy constraints and b) fuzzy goals.

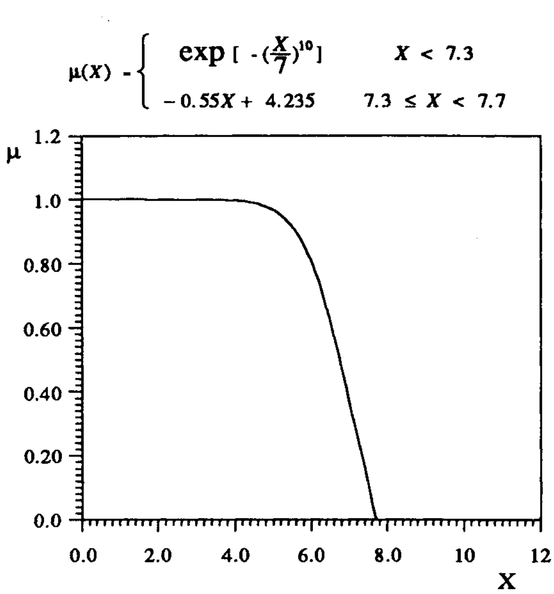

Fig. 3 Example of modified Weibull membership function.

where f^u and f^l are determined from the results of ordinary optimization under severe and loose conditions, respectively. Figure 2 shows a shape of $\mu_{\tilde{g}}$ and $\mu_{\tilde{f}}$.

Microscopic defects such as dislocation, vacancies, and inclusion are coalesced and grow as microcracks in structural materials through the various loading histories. These defects and cracks cause a decrease in stiffness and weakening in strength. The detailed process of the deterioration of the mechanical properties of a material is not so simple or clear. In macroscopic sense, however, it can be assumed that microcracks are continuously distributed in a material.

The measure of distribution of microcracks in materials is chosen as an internal state variable which indicates the change of load-bearing capacity of a material in continuum damage mechanics.[13-17] The fact that the load-bearing capacity or strength of materials changes means that the failure of a material has a fuzzy nature, so the membership function for failure is related to the damage variable. In this paper, in addition to the linear membership function, the exponential-style membership function is defined as a simple form of damage variable with modifications from the computational results of continuum damage mechanics.[16,17]

$$\mu(X) = \begin{cases} \exp\left[-(X/\beta)^{\alpha}\right] & X < X_t \\ \text{linear function} & X \geq X_t \end{cases} \tag{7}$$

This function will be called a modified Weibull membership function since it is similar form to Weibull distribution in probability theory. Figure 3 shows an example of the modified Weibull membership function.

Finite Element Analysis and Failure Consideration

To impose failure and displacement constraints on the laminated composite plates, it is necessary to calculate the stresses in each lamina and the displacements of the plate. To compute the stresses in each lamina and the displacements of the composite plate, a finite element program called COMFA (COMposite FAilure) was developed. COMFA is based on small-displacement plate theory with transverse shear effect. Linear transverse shear deformation is assumed. Stress equilibrium equations and the associated mechanical boundary conditions are derived from the principle of virtual work. Stress-strain constitutive relations are based on the assumptions that the laminas have transversely isotropic property and are perfectly bonded to each other. The isoparametric finite elements are used and, to avoid shear locking, the selective reduced integration scheme is used.

There are many failure criteria for an anisotropic material, such as maximum stress theory, maximum strain theory, Tsai-Hill theory, Hoffman theory, Tsai-Wu theory, and Hashin theory.[18] Of these criteria, maximum stress theory, Tsai-Wu criteria, and Hashin criteria are implemented in COMFA. In the example computation, maximum stress theory is used. The maximum stress theory says that the stresses in the principal material directions must be less than the respective strengths to avoid failure.[19] This is written as

$$\sigma_1 < X, \qquad \sigma_2 < Y, \qquad \sigma_{12} < S \tag{8}$$

Stresses are calculated at 18 Gauss points for each lamina per one element (9 points over the rectangular plane and 2 points along the thickness direction).

Numerical Procedure

Figure 4 shows the procedure of fuzzy optimization. First, the conventional optimizations are performed for the loose condition g^u and the severe condition g^l of the constraints. This procedure is necessary to find the upper and the lower bounds of the objective function from which fuzzy goals are set. For the next step, the fuzzy formulation (2) is optimized by the conventional optimization technique and optimal values for the maximum satisfaction s^*, optimal design vector X^*, and optimal objective f^* are obtained.

The transformed problems such as Eq. (2) are solved by the augmented Lagrange multiplier method with the quasi-Newton procedure.[20] During the minimizing process, the polynomial interpolation method[20] is used as the one-dimensional search method.

Numerical Examples

Two examples are considered to apply the aforementioned procedures to the optimal design of a composite plate. The symmetric eight-layer plate, simply supported on all edges,

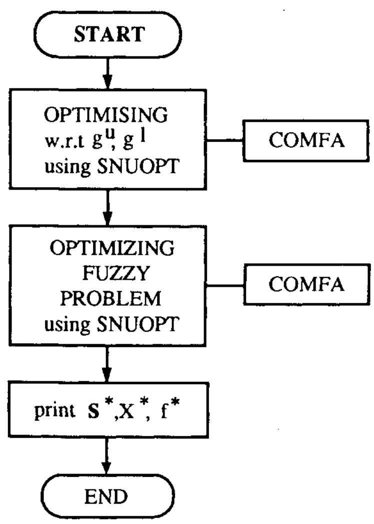

Fig. 4 Procedure of fuzzy optimization.

shown in Fig. 5, is used as a numerical model. Constraints are imposed on the displacement of center point and the strength of laminated composite plate. Table 1 shows the material properties of a carbon fiber reinforced plastic (CFRP) used for the example computation. A 500-kN vertical load is applied at the center of the plate. Starting points in the optimization procedure are the same in all of the examples such that thickness of each lamina is 5 mm and fiber direction of each layer is 90 deg.

Example 1

In the first example, the design variables are the fiber directions and the thicknesses of the laminas. Here, the two-level optimization procedure is used. Linear membership functions are assumed and the severe and the loose conditions in the constraints are given in Table 2. To determine the fuzzy goal, two conventional optimization problems with the severe condition and the loose condition are solved, respectively.

Find X which minimizes $f(X)$ subject to

$$u \le \bar{u}^u \quad (\text{or } \bar{u}^l)$$

$$\sigma_1 \le X^u \quad (\text{or } X^l)$$

$$\sigma_2 \le Y^u \quad (\text{or } Y^l)$$

$$\sigma_{12} \le S^u \quad (\text{or } S^l)$$

$$X_{\min} \le X \le X_{\max} \qquad (9)$$

where

$$X^t = (t_1, t_2, t_3, t_4, \theta_1, \theta_2, \theta_3, \theta_4)$$

Table 1 Properties of a CFRP

Property	Units	CFRP
E_{11}	GPa	134.00
E_{22}	GPa	9.85
G_{12}, G_{13}	GPa	4.94
G_{23}	GPa	2.47
ν_{12}	GPa	0.3
ρ	kg/m^3	1500.00

Table 2 Severe and loose conditions of the design (Example 1)

	Severe condition	Loose condition
$\bar{u}$, mm	5.00	7.00
X, MPa	1300.00	1400.00
Y, MPa	51.60	70.00
S, MPa	80.00	90.00

The two-level procedure is used to consider the influences of the fiber direction and the thickness separately. The first-level problem is to find the optimal fiber directions of each layer which minimizes the center displacement of the plate and the maximum stresses of the plate for the given load. This multi-objective optimization problem can be transformed into a scalar optimization problem by creating the total objective function $Z(\theta)$ of the form

$$Z(\theta) = q_1(u/\bar{u}) + q_2(\bar{\sigma}_1/X) + q_3(\bar{\sigma}_2/Y) + q_4(\bar{\sigma}_{12}/S) \qquad (10)$$

where $\bar{\sigma}_1$ means the maximum of the stresses σ_1, calculated at 1152 points of the plate (18 points/layer × 8 layers/element × 8 elements) and σ_2, $\bar{\sigma}_{12}$ are also obtained in the same way. In our study, the values $q_1 = 2$, $q_3 = 3$, and $q_2 = q_4 = 1$ are chosen. The bigger value is assigned to q_3 since lamina failure is generally governed by the stress σ_2.

The first-level problem is written as follows:

Find X which minimizes $Z(\theta)$ subject to

$$\theta_{\min} \le \theta \le \theta_{\max} \qquad (11)$$

The second-level problem is to find the thicknesses of the laminas minimizing the weight of the plate with the fiber directions fixed by the computed optimal values of the first-level problem. The results are shown in Table 3.

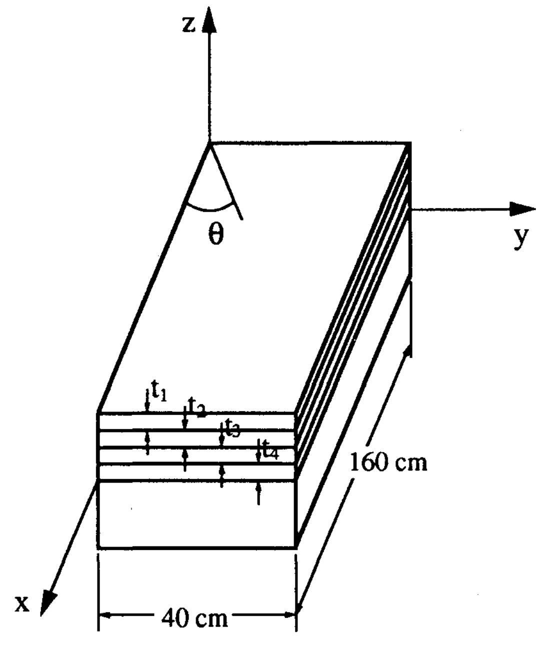

Fig. 5 Numerical model: symmetric eight-layer plate simply supported on all edges.

Table 3a Results of Example 1 (first level)

Maximum satisfaction	Loose condition	Severe condition	Fuzzy
	0.2248		
θ_1, deg	99.2	87.0	79.7
θ_2, deg	25.1	14.8	126.3
θ_3, deg	0.0	149.2	28.3
θ_4, deg	90.1	55.0	105.2
Scalar Obj.	5.58	5.33	—
σ_1, MPa	725.95	512.70	606.10
σ_2, MPa	65.50	47.62	58.77
σ_{12}, MPa	21.32	12.85	18.54
u, mm	7.078	5.000	5.895

Table 3b Results of Example 1 (second level)

Maximum satisfaction	Loose condition	Severe condition	Fuzzy
			0.5525
t_1, mm	6.841	7.475	5.092
t_2, mm	5.849	1.638	5.369
t_3, mm	1.241	6.179	6.033
t_4, mm	5.609	7.560	4.531
Weight, kg	37.517	43.877	40.365
σ_1, MPa	717.67	506.64	606.18
σ_2, MPa	67.50	49.34	58.74
σ_{12}, MPa	20.97	15.08	18.54
u, mm	7.039	5.000	5.894

Now, let us optimize the problem in a fuzzy environment. The first-level problem can be transformed into the following:

Find X which maximizes s_1 subject to

$$s_1 \le 1 - (\sigma_1 - 1.3\times 10^9)/0.1\times 10^9$$
$$s_1 \le 1 - (\sigma_2 - 0.0516\times 10^9)/0.0184\times 10^9$$
$$s_1 \le 1 - (\sigma_{12} - 0.08\times 10^9)/0.01\times 10^9$$
$$s_1 \le 1 - (u - 5.0)/2$$
$$X_{\min} \le X \le X_{\max} \tag{12}$$

where

$$X^t = (\theta_1, \theta_2, \theta_3, \theta_4, s_1)$$

and

$$0.0 \le s_1 \le 1.0$$

The second-level is written as follows:

Find X which maximizes s_2 subject to

$$s_2 \le 1 - (\sigma_1 - 1.3\times 10^9)/0.1\times 10^9$$
$$s_2 \le 1 - (\sigma_2 - 0.0516\times 10^9)/0.0184\times 10^9$$
$$s_2 \le 1 - (\sigma_{12} - 0.08\times 10^9)/0.01\times 10^9$$
$$s_2 \le 1 - (u - 5.0)/2$$
$$s_2 \le 1 - (f - 37.517)/6.360$$
$$X_{\min} \le X \le X_{\max} \tag{13}$$

where

$$X^t = (t_1, t_2, t_3, t_4, s_2)$$

and

$$0.0 \le s_2 \le 1.0$$

The solutions for these problems are obtained in the second iteration with the tolerance $|s^* - s| < 0.001$ and are shown in Table 3. The value of the satisfaction of the first level is an intermediate one since it depends on the value of lamina thicknesses also. The final satisfaction s^* should be the same even though a set of the different initial starting thicknesses of the plate in the first level is used. But the final satisfaction will be different for the choice of the membership functions. Therefore, it is noted that the appropriate derivation of the membership function is important in the fuzzy optimization. To see this, let us use another membership function which has more physical meaning.

Example 2

To show how important is the selection of the membership function, the preceding plate problem is revisited with different conditions. Lamina angles of the plate are fixed as [90/45/ − 45/0 deg]$_s$ for easy comparison. This means that the single-level optimization technique is needed. The severe and the loose conditions are given in Table 4. The modified Weibull membership functions for the failure constraints are assumed as follows:

Table 4 Severe and loose conditions of the design (Example 2)

	Severe condition	Loose condition
$\bar{u}$, mm	5.00	7.00
X, MPa	1135.00	1420.00
Y, MPa	54.00	68.00
S, MPa	72.00	90.50

Table 5 Results of Example 2

	Loose condition	Severe condition	Linear	Modified Weibull
Maximum satisfaction			0.541	0.563
t_1, mm	4.642	5.613	5.255	5.267
t_2, mm	2.916	3.467	3.255	3.332
t_3, mm	6.463	8.709	7.916	8.599
t_4, mm	5.412	4.999	4.600	3.907
Weight, kg	37.257	43.789	40.368	40.493
σ_1, MPa	670.42	478.24	565.13	561.45
σ_2, MPa	67.98	50.62	59.31	58.96
σ_{12}, MPa	21.41	15.96	18.52	18.45
u, mm	7.041	5.000	5.920	5.872

1) Longitudinal strength:

$$\mu_{\tilde{X}} = \begin{cases} \exp\left[-\left(\dfrac{X}{1.370\times 10^6}\right)^{24.774}\right] & X < 1.4\times 10^9 \\ -9.18\times 10^{-9}X + 13.0356 & 1.4\times 10^9 \le X \le 1.42\times 10^9 \end{cases} \tag{14}$$

2) Transverse strength:

$$\mu_{\tilde{Y}} = \begin{cases} \exp\left[-\left(\dfrac{Y}{65\times 10^6}\right)^{24.774}\right] & Y < 6.7\times 10^7 \\ -1.733\times 10^{-7}Y + 11.7844 & 6.7\times 10^7 \le Y \le 6.8\times 10^7 \end{cases} \tag{15}$$

3) Shear strength:

$$\mu_{\tilde{S}} = \begin{cases} \exp\left[-\left(\dfrac{S}{87\times 10^6}\right)^{24.774}\right] & S < 8.8\times 10^7 \\ -1.0608\times 10^{-7}S + 9.60024 & 8.8\times 10^7 \le S \le 9.05\times 10^7 \end{cases} \tag{16}$$

To set the fuzzy goal, two crisp optimization problems are solved in the same manner as the previous example. The results are shown in Table 5.

Then, the fuzzy optimization problem with the linear membership functions is the following:

Find X which maximizes s subject to

$$s \le 1 - (\sigma_1 - 1.135\times 10^9)/0.285\times 10^9$$
$$s \le 1 - (\sigma_2 - 0.054\times 10^9)/0.014\times 10^9$$
$$s \le 1 - (\sigma_{12} - 0.072\times 10^9)/0.0185\times 10^9$$
$$s \le 1 - (u - 5.0)/2$$
$$s \le 1 - (f - 37.257)/6.532$$
$$X_{\min} \le X \le X_{\max} \tag{17}$$

where

$$X^t(t_1, t_2, t_3, t_4, s)$$

With the modified Weibull membership functions, the problem can be written as follows:

Find X which maximizes s subject to

$$s \le \mu_{\tilde{X}}(X)$$

$$s \le \mu_{\tilde{Y}}(X)$$

$$s \le \mu_{\tilde{S}}(X)$$

$$s \le 1 - (u - 5.0)/2$$

$$s \le 1 - (f - 37.257)/6.532$$

$$X_{\min} \le X \le X_{\max} \tag{18}$$

where

$$X^t = (t_1, t_2, t_3, t_4, s)$$

The results of these two formulations are shown in Table 5. It is noted that different final satisfactions are obtained since the modified Weibull membership function is smoother than the linear membership function at the severe side of the constraint. It can be concluded that the useful formulation of the fuzzy optimization is hinged on the appropriate membership function which should be selected from sound physical observations, experimental data bases, and designer's experience, etc.

Conclusions

The optimal design of laminated composite plates in a fuzzy environment is presented. The design goal is to minimize the weight of the plate. The constraints are imposed on the displacement of center point and lamina failure of the plate. Two-level optimization in a fuzzy environment is presented in a more elegant form and applied to the fuzzy optimal design of laminated composite plates. Using the finite element method, the displacements of the plate and the stresses of each lamina are calculated. The augmented Lagrange multiplier method with the quasi-Newton method is used to solve the conventional problems transformed from the fuzzy optimization problem. Two examples are solved to demonstrate the procedure and to highlight the importance of the membership function.

The choice of the membership function is the most important factor of the fuzzy optimization procedure. It may depend on the designer's experience, experimental data bases, and sound physical observations, etc. It is shown that the membership function for failure criteria can be derived from continuum damage mechanics and is a function of the damage variable. The modified Weibull membership function characterizes the fuzziness of failure.

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